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Modeling Solid-Fuel Ramjet Combustion, Including Radiation to the Fuel Surface

Michael E. Metochianakis* and David W. Netzer†
Naval Postgraduate School, Monterey, California

Nomenclature

B = mass transfer number (or "blowing parameter")
G = mass flux
$\tilde{h}$ = stagnation enthalpy
m = mass fraction
$\dot{m}''$ = mass flux
n = number density of particulates in the flame zone
$\dot{q}''$ = heat flux
T = temperature
w_{fu} = mass of unburned fuel in the boundary layer
α = empirical constant
δ = thickness of the combustion zone
ϵ_w = emissivity and absorptivity of the wall
ϵ_g = emissivity of the flame zone
λ = empirical constant
ρ = density
σ = Stefan-Boltzmann constant

Subscripts

con = convection
f = flame, fuel
fu = fuel grain
p = near wall node
rad = radiation
w = wall

Introduction

THE solid-fuel ramjet (SFRJ) most often consists of a solid-fuel grain which provides the walls for the combustion chamber.[1] A sudden expansion at the air inlet end of the combustor can be used to provide flame stabilization in the SFRJ. Downstream of the flow reattachment a turbulent boundary layer develops and includes a diffusion-controlled flame between the fuel-rich zone near the wall and the oxygen-rich central core. Due to that diffusion flame, heat is transferred by convection and radiation to the solid surface, causing vaporization of the fuel.

At the Naval Postgraduate School both mathematical modeling[1-3] and experimental efforts[4-9] have been conducted to determine the effects of design and operational variables on the obtainable performance.

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*Captain, Hellenic Air Force.

†Professor, Department of Aeronautics. Member AIAA.

The computer model simulation of the SFRJ combustion process has evolved from an original stream-function vorticity formulation[1] to a primitive-variable (pressure, velocity) model which includes an aft mixing chamber.[3] These axisymmetric models have not included radiative heat transfer to the fuel surface. This has limited the utility of the model, since many all-hydrocarbon fuels produce significant amounts of radiative transfer through the generation of carbon particulates in the flame zone.

Model Overview

The CHAMPION 2/E/FIX computer program developed by Pun and Spalding[10-12] was used as the basis for the primitive variable model. The combustion was assumed to be steady, subsonic, recirculating, axisymmetric, and mixing limited and to have constant specific heat. Details of the model assumptions and the method of solution of the finite-difference equations have been presented in Ref. 3.

The regression rate of the fuel is proportional to the total heat transfer to the surface (Fig. 1). This heat transfer consists of two interdependent parts, convection and radiation.

The convective heat transfer depends on the flow conditions within the port of the fuel grain, primarily upon the air temperature and the air mass flow per unit area.

The radiant energy to the fuel surface can be approximated by

$$\dot{q}''_{\mathrm{rad}} = \sigma\epsilon_w(\epsilon_g T_f^4 - T_w^4) \tag{1}$$

The emissivity of the flame zone can be written[14]

$$\epsilon_g = 1 - e^{-\alpha n\delta} \tag{2}$$

In this application, only the dominant effect of radiation from the carbon particles within the flame zone was considered. It was assumed that the gases between the flame and the wall were transparent and that gas-phase radiation was negligible. It was further assumed that

$$n_{\mathrm{flame}} \sim n_{\substack{\mathrm{boundary}\\ \mathrm{layer}}} \sim \frac{\text{Mass of unburned fuel}}{\text{Vol}} \equiv \frac{w_{fu}}{\text{Vol}} \sim \rho m_{fu} \tag{3}$$

The flame zone thickness δ has been found to be proportional to the square root of the port diameter, but for a given geometry it was found to be almost constant.[14] When these approximations are incorporated into Eq. (2) the following expression is obtained:

$$\epsilon_g \equiv 1 - e^{(-w_{fu}/\mathrm{Vol})/\lambda} \tag{4}$$

To estimate the mass of unburned fuel in the boundary layer at a particular axial location, a summation was made over control volumes of the finite-difference grid from the fuel surface to the axis of symmetry.

The mass transfer number without radiation must be modified to account for thermal radiation.[13] Thus,

$$B = \frac{\tilde{h}_p - \tilde{h}_w}{\tilde{h}_w - \tilde{h}_{fg} - (\dot{q}''_{\mathrm{rad}}/\dot{m}'')} \tag{5}$$

In this investigation, T_w was taken to be the reference temperature for enthalpy[13] and therefore $\tilde{h}_w = 0$.

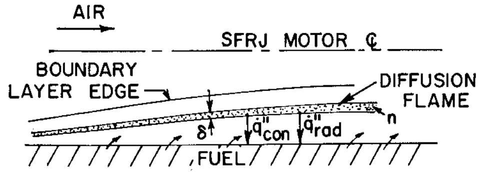

Fig. 1 Schematic of SFRJ boundary-layer combustion process.

The same solution procedure used for the simple convection model[3] was followed in this investigation.

Results and Discussion

For the Plexiglas (PMM)-O_2 hybrid combustion system (which has only small radiation effects) a typical value for ϵ_g is 0.02 and the radiative heat flux to the fuel grain is approximately 15% of the convective flux.[15] Using these values, the empirical constant λ in Eq. (4) is approximately three.

For some fuels it has been found experimentally that increased combustor pressure increases the average regression rate. Small variations in fuel regression rate with pressure at fixed air mass flux cannot be explained in terms of the convection heat transfer because it normally depends on only the mass flux. Pressure changes that result from decreasing the nozzle throat size necessarily increase $\dot{q}''_{\rm rad}$. In some combustion systems, finite-rate kinetics can also result in significant pressure effects.

Fuels that generate large amounts of carbon [such as hydroxy terminated polybutadiene (HTPB)] also result in different regression rate patterns compared with the patterns obtained using PMM. The main difference is that in the HTPB-type systems the regression rates increase with axial distance at the aft end of the grain. One possible cause of such an effect is that as the reacting flow gets hotter and accelerates the convective heat transfer is increased. However, the computer calculations[3] indicated that this was not a dominant effect. Therefore, $\dot{q}''_{\rm rad}$ was considered to be the reason for this behavior.

Figure 2 shows the regression profiles obtained for the two cases: with and without radiation heat transfer to the fuel surface. The flow reattachment point was not affected by incorporation of radiation into the model. Radiation increased the regression rates downstream of reattachment, resulting in a regression profile in better agreement with experiment.[3] The amount of radiation in the model depends strongly upon the empirical constant λ. This value is selected to match experimental data for one fuel and one set of test conditions. More data are required to determine whether or not one value of λ can be used for varying test conditions with one fuel.

The average regression rate of Plexiglas has been reported to vary significantly with chamber pressure.[4] Closer examination of that data revealed that for fixed geometry and mass flux the pressure effect was quite weak. The present model predicted the average regression rate to vary as $p_c^{0.06}$.

It has been found experimentally that the regression rate of Plexiglas varies with the air mass flux as G^y. For the three cases considered here (inlet air velocities of 158, 197, and 236 m/s), the constant y was predicted to be between 0.23 and 0.28, while Mady et al.[6] experimentally found it to be approximately 0.38. Therefore, the model with radiation appears to correctly predict the nature of the change in convective heat flux to the fuel surface with airflow rate, but to underestimate the magnitude.

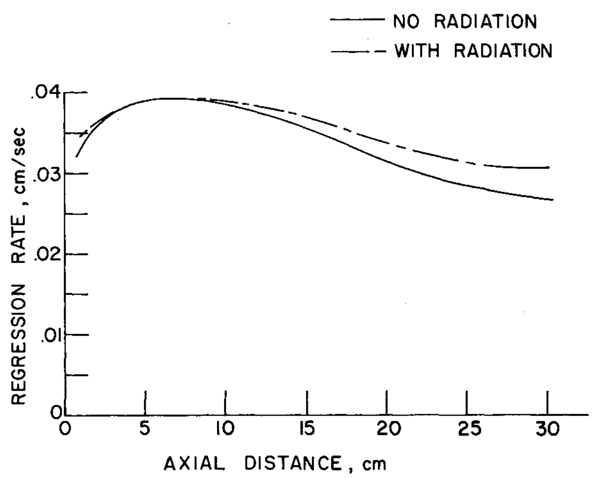

Fig. 2 Effect of radiation on the predicted fuel regression rates.

For fuel systems that produce larger amounts of carbon particulates than the Plexiglas, a lower value of the empirical constant λ resulted in higher regression rates near the aft end of the fuel grain. This results from the increasing amounts of unburned fuel below the flame with increasing axial distance and is in general agreement with observed experimental behavior.[3,9]

Conclusions

The addition of radiative heat transfer to the primitive variable computer model resulted in improved agreement with experimentally obtained fuel regression rate profiles. The effects of varying chamber pressure and air mass flux appear to be qualitatively correct but additional data are required to further validate the model.

Acknowledgment

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